Financial Liberalization, Boom-Bust Cycles and Production Efficiency

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April 2010 (first version: March 2009)
JEL Classification No.F34, F36, F43, O41
1 Introduction

Is it that factors that have been a source of financial fragility have also contributed to financial deepening and growth? If so, is there a sense in which the lending booms and associated risk taking that follow financial liberalization help improve the efficiency of resource allocation? And, should taxpayers be willing to foot the bill associated with the bailouts that typically are granted during crises—i.e., are such bailout policies ex-ante desirable? Finally, should financial liberalization entail regulatory limits on the types of liabilities that can be issued or should laissez-faire reign?

In order to address these issues one should consider three empirical regularities that cut across countries. First, although crises have been costly, countries that have liberalized financially, experienced booms and crises have been among the fastest growing countries. By contrast, non-liberalized countries in which credit growth has been smooth have exhibited low growth rates. Second, there are—implicit and explicit—guarantees to bailout lenders during systemic crises. Third, crises and the lending booms that precede them do not affect all sectors in the economy homogeneously. Rather there is a sharp sectorial asymmetry that cuts across episodes: typically during the boom one sector—real estate or more generally nontradables—grows abnormally fast relative to the rest of the economy, but crashes more severely during a crisis and subsequently suffers a greater decline during the credit crunch.

Because of the empirical importance of sectorial asymmetries, one-sector representative agent frameworks are not appropriate to analyze the financial liberalization policies that tend to generate boom-bust cycles. In this paper we consider a two-sector model with asymmetric financial opportunities that reproduces the sectorial empirical regularities of boom-bust cycles, and provides a framework to analyze the trade-offs between stability, growth and efficiency.

The argument relies on how financing constraints in one sector affect the performance of the whole economy, and on how systemic bailout guarantees influence financing decisions. We consider an economy where a financially constrained sector—the N-sector—produces inputs for the entire economy. In this environment, financing constraints in the N-sector generate a bottleneck that limits the supply of inputs for the other sectors and thus impacts negatively the growth performance and the production efficiency of the economy as a whole. Financial liberalization allows for new financing instruments, which leads to a relaxation of the constraints faced by the N-sector. However, and this is key, the new instruments generate new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into a risky one. Our framework provides an internally consistent mechanism under which such transformation emerges, and helps understand how it can enhance long-run growth and production efficiency even though occasional crises

1 Bekaert, et.al. (2001) and Ranciere et.al. (2008).
2 This sectorial asymmetry is associated with currency mismatch in balance sheets (i.e., while N-sector assets are denominated in the N-price, the liabilities are not), as well as with sharp appreciation spells which are punctuated by dramatic depreciations (in asset prices and real exchange rates). As a result crisis costs are concentrated in the N-sector as many agents in this sector go bust due to balance sheet effects. For instance, recently foreign currency loans have funded a real estate boom in Baltic and Eastern European countries (Ranciere, Tornell and Vamvakidis (2010)); in the US a real estate boom was financed by liabilities indexed to the CPI, but real estate revenues were indexed to housing prices (Ranciere and Tornell (2009)); foreign currency loans financed consumption and investment booms preceding the Asian and Tequila crises (Tornell and Westermann (2002), (2005)).
occur during which the N-sector suffers the costs associated with widespread bankruptcies. It also helps us understand why financial liberalization should be accompanied by financial regulation, as the indiscriminate use of option-like liabilities—such as CDSs—can be harmful.

In our model economy the T-sector produces a final consumption good, and the N-sector produces an intermediate input which is used in both sectors. While the T-sector has access to perfect capital markets, the N-sector faces contract enforceability problems. As a result, N-sector investment is constrained by its internal funds. The economy has also systemic bailout guarantees, which insure lenders only against systemic crises, but not against an idiosyncratic default.

We consider three financial regulatory regimes: repression, liberalization and laissez-faire. Under financial repression N-agents can only issue standard debt contracts—i.e., liabilities that promise to repay in all states of the world—whose promised repayment is indexed to the N-price. Under financial liberalization, currency mismatch is allowed: promised repayments need not be indexed to the N-price. However, regulation only allows standard debt contracts. Finally, under laissez-faire—i.e., an anything goes regime—agents can also issue liabilities that promise to repay only in one state of the world—for instance, the sale of options and credit default swaps.

In order to address the growth-stability trade-off the model captures two costs typically associated with crises: bankruptcy and financial distress costs. Bankruptcy costs are static and derive from the severe real depreciation that leads to firesales and bankrupts N-sector firms with currency mismatch on their balance sheets. Financial distress costs are dynamic and derive from the resultant collapse in internal funds and the reduction in risk taking in the aftermath of crisis, which depresses new credit and investment, hampering growth.

Since there are no exogenous shocks, under financial repression there exists only one equilibrium where insolvencies and crises never occur. If there are significant contract enforceability problems, along this safe equilibrium the N-sector exhibits low growth because its investment is constrained by its cash flow. Since N-goods serve as intermediate inputs for both sectors, the N-sector constrains the long-run growth of the T-sector and that of GDP: there is a bottleneck that constrains aggregate growth. In contrast, under financial liberalization, there is an additional risky equilibrium in which real exchange rate risk arises endogenously and N-firms find it optimal to take on insolvency risk in the form of currency mismatch. This risky behavior relaxes borrowing constraints, increases investment, alleviates the bottleneck and allows both sectors to grow faster. However, it also generates financial fragility, as a shift in expectations can cause a sharp fall in the price of N-goods relative to T-goods, bankrupt N-firms and land the economy in a crisis.

Our first result is that if there is a bottleneck, a financially fragile economy will on average grow faster than a safe economy even if bankruptcy costs are large, provided that the dynamic crisis costs are not too severe. This result follows from the fact that crises must be rare events in order for credit risk to be profitable for individual borrowers that must risk their own equity, and from the fact that not all the bankruptcy losses experienced by the N-sector during crises are aggregate deadweight losses. This is because firesales redistribute resources away from the N-sector towards the T-sector. The financial distress costs of crises can be far more significant than bankruptcy costs because they spread dynamically: the decline in internal funds and the reduction in risk taking translate into a depressed leverage and investment in the
N-sector that reduces growth.

Because both sectors compete every period for the available supply of N-inputs, when contract enforceability problems are very severe, the N-sector attains low leverage and commands only a small share of N-inputs. This results in a socially inefficient low growth path: a central planner would increase the N-sector investment share to attain the Pareto optimal allocation.

In a decentralized economy, the first best can be attained by reducing the agency problems that generate the financing constraints. However, if such a reform is not feasible, financial liberalization and the credit risk that it entails, may be seen as an alternative way to improve the allocation despite financial fragility. Our second result is that when contract enforceability problems are severe, but not too severe, credit risk brings the allocation nearer to the Pareto optimal level and increases the present value of consumption that the economy can attain, even if we take into account bankruptcy costs, as long as the dynamic crisis costs are not too large.

The existence of the risky equilibrium depends on systemic bailout guarantees. Since these guarantees are funded by domestic taxation the question arises as to whether such a policy can be implemented. We show that the bailout guarantee is fundable through taxes on the T-sector. Furthermore, if there is a bottleneck to begin with, and N-inputs are intensively used in T-production, the T-sector will find it profitable to fund the fiscal cost of the guarantees. The funding of the guarantees actually effects a redistribution from the non-constrained T-sector to the constrained N-sector. This redistribution is to the mutual benefit of both sectors because over the long-run T-production enjoys more abundant N-inputs, and its growth rate increases: the bottleneck is eased. Thus, even those who bear the costs of crises may be willing to pay their price.

The efficiency benefits described above rely on the fact that the increase in leverage occurs without loosing the discipline that comes from the requirement that borrowers must risk their own equity. In the model this discipline comes about by limiting external finance to standard debt contracts under which agents must repay in all states to avoid bankruptcy. This discipline is missing in a laissez-faire regime under which agents can also issue option-like liabilities. Since we allow for systemic bailout guarantees, even borrowers with no profitable projects could issue option-like liabilities that promise to repay only in the crisis state, and lenders would be willing to buy them. Such liabilities permit agents to attain an unreasonable high leverage without risking their own equity, and might induce equilibria with diversion scams. Thus, in the presence of implicit or explicit–bailout guarantees, a laissez faire regime might hamper growth and production efficiency.

This paper is motivated by the empirical findings presented in Ranciere, Tornell and Westermann (2008), which show that systemic crisis risk is associated with higher long-run growth. Thailand and India, over 1980-2001, are contrasting examples of a steep but crisis prone growth path and a slow but safe growth path. Thailand has experienced lending booms and crises, while India has pursued a safe growth path for credit (see Figure 1). GDP per-capita grew by only 99% between 1980 and 2001 in India, whereas Thailand’s GDP per-capita grew by 148%, despite having experienced a major crisis. Revealingly, India has recently evolved towards a more financially liberalized regime and has experienced rapid credit growth. Meanwhile, Thailand has seemingly reverted to a safe credit growth path after having suffered a deep credit crunch in the aftermath of a crisis. Between 2001 and 2008 the per-capita GDP gap between India and Thailand has
been significantly narrowed.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 analyses long-run growth. Section 4 analyzes production efficiency under financial repression and financial liberalization. Section 4.4 considers laissez faire regime. Section 5 relates our paper to the literature. Section 6 concludes.

**Figure 1: Safe vs. Risky Growth Path**

2 Model

We consider an infinite horizon endogenous growth model of a two-sector small open economy with credit market imperfections that embeds the credit market game of Schneider and Tornell (2004).

There are two goods: a tradable (T) good, which is the consumption good, and a nontradables (N) good, which is used as an input in the production of both goods. We will denote the relative price of N-goods by \( p_t = p_N^t / p_T^t \).

There are no exogenous shocks. The only source of uncertainty is endogenous price risk: in equilibrium \( p_{t+1} \) may equal \( p_{t+1} \) with probability \( u_{t+1} \) or \( \overline{p}_{t+1} \) with probability \( 1 - u_{t+1} \). The probability \( u_{t+1} \) may equal either 1 or \( u \), and this is known at \( t \).

There are competitive risk neutral international investors whose cost of funds equals the world interest rate \( r \). These investors lend any amount as long as they are promised an expected payoff of \( 1 + r \). They also issue default-free bonds: an N-bond and a T-bond. The T-bond pays \( 1 + r \) next period, while the N-bond pays \( (1 + r^n) p_{t+1} \). The existence of risk neutral deep-pocket investors implies that uncovered interest parity will hold in any equilibrium

\[
(1 + r^n) p_{t+1} = 1 + r, \quad \text{where} \quad p_{t+1} := u_{t+1} T_{t+1} + (1 - u_{t+1}) \overline{T}_{t+1}
\]

There is a continuum, of measure one, of competitive firms that produce the T-good using as input the

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3 In an international setup \( p_t \) is the inverse of the real exchange rate.
N-good \( (d_t) \) and a non-reproducible factor \( (I_t^T) \). The representative T-firm maximizes profits taking as given the price of N-goods \( (p_t) \) and the price of the non-reproducible factor \( (v_t^T) \):

\[
\max_{\{d_{t+j}, I_{t+j}^T\}_{j=0}^\infty} \left[ y_{t+j} - p_{t+j}d_{t+j} - v_{t+j}^T I_{t+j}^T \right], \quad y_{t+j} = a_{t+j}d_{t+j}^\alpha (I_{t+j}^T)^{1-\alpha}, \quad \alpha \in (0, 1)
\]

There is a continuum, of measure one, of consumers. The representative consumer is infinitely lived, consumes only T-goods, and is endowed with one unit of the non-reproducible factor, which he supplies inelastically \( (I_t^T = 1) \). Furthermore, he can buy and sell any amount of the two default-free bonds described above. He solves the following problem.

\[
\max_{\{c_{t+j}\}_{j=0}^\infty} E_t \sum_{j=0}^{\infty} \delta^j U(c_{t+j}), \quad \text{st.} \quad E_t \sum_{j=0}^{\infty} \delta^j [c_{t+j} - v_t^T T_{t+j}] \leq 0, \quad \delta := \frac{1}{1+r}
\]

where \( T_t \) is the tax that will finance the bailouts, as we describe below.

There is a continuum, of measure one, of productive firms that produce N-goods using entrepreneurial labor \( (l_t) \), and capital \( (k_t) \). Capital consists of N-goods invested during the previous period \( (I_{t-1}) \), which fully depreciates after one period. The production function is

\[
q_t = \Theta_t l_t^\beta k_t^{1-\beta}, \quad \Theta_t =: \theta k_t^{1-\beta}, \quad k_t = I_{t-1}, \quad \beta \in (0, 1)
\]

The technological parameter \( \Theta_t \) embodies an external effect, where \( \overline{k}_t \) is the average N-sector capital, that each firm takes as given.

The investable funds of an N-firm consist of its internal funds \( w_t \) plus the liabilities \( B_t \) it issues. These investable funds can be used to buy default-free bonds \( (s_t, s_t^n) \) or N-goods \( (p_t I_t) \) in order to produce N-goods in the following period. Since \( B_t \) is measured in T-goods, the time \( t \) budget constraint and time \( t+1 \) profits are, respectively

\[
p_t I_t + s_t + s_t^n = w_t + B_t
\]

\[
\pi(p_{t+1}) = p_{t+1}q_{t+1} + (1+r)s_t + p_{t+1}(1+r_t^n)s_t^n - v_{t+1}l_{t+1} - L_{t+1}
\]

\( L_{t+1} \) is the next period’s promised repayment, which we describe below.

N-firms are run by overlapping generations of entrepreneurs who live for two periods and consume only tradables in the second period of their life. At the beginning of time \( t \) a young entrepreneur supplies inelastically one unit of labor \( (l_t = 1) \) and receives a wage \( v_t \). At the end of time \( t \) she takes control of the firm and makes investment and financing decisions. The cash flow of the firm equals the entrepreneur’s wage:

\[ w_t = v_t. \]

Finally, there is a set of non-productive firms that can issue liabilities but have access to a production technology. This set of firms will become relevant when we consider the laissez faire regime.

N-sector financing is subject to two credit market imperfections: contract enforceability problems and systemic bailout guarantees that cover lenders against systemic crises. The former might give rise to borrowing constraints in equilibrium, while the latter might induce firms to undertake insolvency risk through currency mismatch.

Contract Enforceability Problems. Entrepreneurs cannot commit to repay their liabilities: if at time \( t \) the entrepreneur incurs a non-pecuniary cost \( h|w_t + B_t| \), then at \( t+1 \) she will be able to divert all the returns provided the firm is solvent (i.e., \( \pi(p_{t+1}) \geq 0 \)).
**Systemic Bailout Guarantees.** If more than 50% of firms become insolvent (i.e., \( \pi(p_t) < 0 \)), a bailout agency pays lenders the outstanding liabilities of each defaulting firm up to an amount \( G_t \) per firm. The guarantee applies to all types of liabilities. Since there are bankruptcy costs, the bailout agency recuperates a share \( \mu \) of the insolvent firms’ revenues. The remainder is financed by lump-sum taxes on consumers.

We consider three regulatory regimes that determines the set of liabilities that debtors can issue. First, a "financially repressed regime" under which N-firms can only issue one-period bonds that promise to repay in N-goods: \( L_{t+1} = p_{t+1}(1 + \rho_{t+1}^n)b^n_t \), where \( \rho_{t+1}^n \) is the nominal interest rate and \( b^n_t \) is the amount borrowed-denominated in T-goods. Second, there is a "financially liberalized regime" under which a firm is free to issue T-debt \( (b_t) \) and N-debt \( (b^n_t) \), but can only issue standard one-period debt contracts—i.e., it must promise to repay in both states of world: crisis and no-crisis. Under this regime, the promised debt repayment in both states is

\[
L_{t+1} = (1 + \rho_{t+1})b_t + p_{t+1}(1 + \rho_{t+1}^n)b^n_t
\]  

(7)

Under this regime firms can take on currency mismatch: if \( b_t > 0 \) and \( p_{t+1} = p^n_{t+1} \), at \( t+1 \) revenues will fall more steeply that debt repayments. Finally, there is an "laissez-faire regime" under which firms can take on currency mismatch and are not restricted to issue standard debt contracts, but can also issue liabilities that promise to repay in only one state of the world. They can thus issue liabilities that promise to repay next period \( L_{t+1} = (1 + \rho_{t+1}^b)B_t \) if \( p_{t+1} = p^n_{t+1} \), and zero if \( p_{t+1} = p^b_{t+1} \). This payoff structure characterizes instruments such as out-of-the money options and credit default swaps.4

The goal of every entrepreneur is to maximize next period’s expected profits net of diversion costs. Since guarantees are systemic, the decisions of entrepreneurs are interdependent. They are determined in the following credit market game, which is similar to that considered by Schneider and Tornell (2004). During each period \( t \), taking prices as given, every young entrepreneur proposes a plan \( P_t = (I_t, s_t^n, b_t^n, b^n_t, L_{t+1}) \) that satisfies budget constraint (5). Lenders then decide whether to fund these plans. Finally, funded young entrepreneurs make investment and diversion decisions.

Payoffs are determined at \( t + 1 \). Consider first plans that do not lead to diversion. If the firm is solvent \( (\pi(p_{t+1}) \geq 0) \), the old entrepreneur pays \( v_{t+1} \) to the young entrepreneur and \( L_{t+1} \) to lenders. She then consumes the profit \( v_{t+1} = \pi(p_{t+1}) \). In contrast, if the firm is insolvent \( (\pi(p_{t+1}) < 0) \), young entrepreneurs receive \( \mu w p_{t+1} q t+1 \) (\( \mu w < 1 - \beta \)), lenders receive the bailout if any is granted, and old entrepreneurs get nothing. Consider next plans that entail a diversion scheme. If the firm is solvent, the old entrepreneur gets \( \beta p_{t+1} q t+1 \), young entrepreneurs get [1 - \( \beta \)]\( p_{t+1} q t+1 \) and lenders receive the bailout if any is granted. Under insolvency entrepreneurs get nothing and lenders receive the bailout if any is granted. The problem of a young entrepreneur is then to choose an investment plan \( P_t \) and diversion strategy \( \eta_t \) that solves:

\[
\max_{P_t, \eta_t} E_t \left[ \xi_{t+1} \left\{ p_{t+1} q_{t+1} + (1 + r) s_t + p_{t+1} (1 + r^n) s^n_t - v_{t+1} L_{t+1} - (1 - \eta_t) L_{t+1} \right\} - \eta_t h(w_t + B_t) \right]
\]

subject to (5), where \( \eta_t = 1 \) if the entrepreneur has set up a diversion scheme, and zero otherwise; and \( \xi_{t+1} = 1 \) if \( \pi(p_{t+1}) \geq 0 \), and zero otherwise. The following definition integrates the credit market game with the rest of the economy.

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4These instruments tend to be accounted off the balance sheet of firms and financial institutions. In contrast, standard debt contracts are typically accounted in the balance sheet.
Definition. A symmetric equilibrium is a collection of stochastic processes
\( \{ I_t, s_t, s^n_t, b_t, b^n_t, L_{t+1}, d_t, c_t, y_t, q_t, u_t, p_t, \omega_t, u_t, v_t, v^T_t \} \) such that, (i) given current prices and the distribution of future prices the plan \( (I_t, s_t, s^n_t, b_t, b^n_t, L_{t+1}) \) is determined in a symmetric subgame perfect equilibrium of the credit market game, \( d_t \) maximizes T-firms profits and \( c_t \) maximizes consumers expected utility; (ii) factor markets clear; and (iii) the market for non-tradables clears:
\[
d_t(p_t) + I_t(p_t, I_{t+1}, \omega_{t+1}, u_{t+1}) = q_t(I_{t-1})
\] (8)

To close the model we assume that date zero young entrepreneurs are endowed with \( w_0 = (1 - \beta)p_oq_o \) units of T-goods, while old entrepreneurs are endowed with \( q_o \) units of N-goods and have no debt in the books. Finally, we impose the condition that guarantees are domestically financed through taxation:
\[
E_t \sum_{j=0}^{\infty} \delta^j [1 - \xi_{t+j}] (L_{t+j} + \omega_{t+j} L^n_{t+j} - \mu p_{t+j} q_{t+j} - T_{t+j}) = 0, \quad \mu \in [0, \beta].
\] (9)

2.1 Discussion of the Setup

In our set-up, there is no exogenous source of shocks. In equilibrium fragility will arise from a self-reinforcing mechanism: N-firms find it profitable to issue T-debt in the presence of systemic guarantees and sufficient real exchange rate variability. This variability, in turn, may arise because there is enough T-debt issued by N-firms.

Markets are complete in our framework. Since during each period the real exchange rate can take only two values, the menu of securities allows consumers and firms to hedge all risk.\(^5\) This will allow us to make the point that growth and efficiency gains arise from the undertaking of credit risk, not from consumption smoothing.

The assumption that N-goods are used as inputs is key. First, the use of N-inputs in N-production is necessary for the existence of endogenous insolvency risk. Otherwise, a crisis state, characterized by bankruptcies, resales and the collapse of the price of N-good, would not exist. Second, the use of N-inputs in T-production together with external effects in N-production imply that the N-sector is the source of endogenous growth in the economy. As we shall see, the share of N-output commanded by the N-sector for investment (\( \phi \)) is the key determinant of aggregate growth. Economies with too small equilibrium \( \phi \) experience a bottleneck to growth. Our results will derive from the fact that the undertaking of credit risk—by increasing the mean value of \( \phi \)—may increase production efficiency, and that the T-sector may derive a net benefit from financing the fiscal costs associated with the bailout guarantees that support such equilibrium.\(^6\)

To capture the dynamic and the static effects of crises we have allowed for two types of crisis costs: financial distress—indexed by \( (1 - \beta)/\mu_w \)—and bankruptcy costs—indexed by \( \beta/\mu \). All the equilibria we characterize exist for any \( \mu_w \in (0, 1 - \beta) \) and \( \mu \in [0, \beta] \).

Financing opportunities are asymmetric across sectors. The N-sector is affected by contract enforceability problems that give rise to financing constraints. The T-sector in contrast has perfect access to capital

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5 In particular, N-debt is a perfect hedge for N-sector firms.

6 In contrast, the assumptions that N-goods are not consumed and T-goods are not intermediate inputs are convenient but not essential. If N-goods were consumed, there would a deeper fall in the demand of N-goods when N-firms become insolvent, accentuating the self-fulfilling depreciation that generates crisis.
market. This assumption schematizes a variety of sectoral differences in external financing options. For example, this representation captures the fact that most of the firms in middle-income countries that can access international financial markets are in the T-sector. In contrast, most N-sector firms are dependent on domestic bank credit.\footnote{This is in part because T-firms can either pledge export receivables as collateral, or can get guarantees from closely linked firms. Tornell and Westermann (2003) document sectorial asymmetries as well as systemic guarantees in MECs.}

The agency problem and the two-period lived entrepreneur set-up is considered by Schneider and Tornell (2004). The advantage of this set-up is that one can analyze financial decisions period-by-period. This will allow us to explicitly characterize the stochastic processes of prices and investment. These closed-form solutions are essential to derive the limit distribution of growth rates and establish our efficiency results.

The assumption that bailout guarantees are systemic is essential. If instead, guarantees were unconditional and a bailout were granted whenever a single borrower defaulted, then the guarantees would neutralize the contract enforceability problems and borrowing constraints would not arise in equilibrium.

The three regulatory regimes we consider—repression, liberalization, and laissez faire—are meant to capture in a simple way three regulatory environments that are observed across the world. One in which there is overregulation, credit policies are restrictive and so leverage is low. Another situation in which agents are free to take on risk but there is an efficient prudential regulatory framework that ensures lenders impose strict repayment criteria in their loans. Finally, a situation where agents have the ability to implement scams that exploit bailout guarantees, like the ones used by AIG.

In our setup, bailout are granted only during a systemic crisis, and borrowers must be solvent in order to divert. As we shall see, under standard debt contracts lenders will only fund firms that can repay debt in the good state, and so there is discipline in the lending mechanism under financial liberalization. However, under laissez faire discipline is lost because lenders are willing to fund scams where the borrower pays nothing in the good state, goes bust in the bad state and the taxpayer pays the bill. If the promised amount in the bad state is large enough and the bailout is expected to cover it, the lenders will be willing to fund this scam.\footnote{Notice that these scams are ‘legal’ in the laissez faire regime. They captures the issuance of puts and credit default swaps by firms such as Lehman, AIG in the US and Cemex and Aracruz in emerging markets.}

2.2 Symmetric Equilibria (SE)

We construct SE in two steps. First, we take prices \( p_t \) and the likelihood of crisis \( 1 - u_{t+1} \) as given, and derive the equilibrium at a point in time. We then endogeneize \( p_t \) and \( u_{t+1} \). In order to simplify notation we will set \( a_t = 1 \) in (2).

The representative T-firm maximizes profits, taking goods and factor prices as given. It thus sets \( p_t d_t = \alpha y_t \) and \( v_T^T = (1 - \alpha) y_t \). Since consumers supply inelastically one unit of the non-reproducible factor, equilibrium T-output, consumer’s income and the T-sector demand for N-goods are, respectively:

\[
y_t = d_t^\alpha, \quad v_T^T = [1 - \alpha] y_t, \quad d(p_t) = \left[ \frac{\alpha}{p_t} \right]^{\frac{1}{1-\alpha}} \tag{10}
\]

Since the consumer’s subjective discount rate equals the risk free rate, in each period he consumes a constant
fraction of his expected discounted income:

\[
ct = [1 - \delta]Et \left( \sum_{j=0}^{\infty} \delta^j [(1 - \alpha)yt+j - Tt+j] \right)
\]  

(11)

In any SE the representative N-firm’’s capital \((k_t)\) is equal to aggregate average capital \((\bar{k}_t)\). Thus, (4) implies that N-output equals: \(qt+1 = \theta k_{t+1} = \theta I_t\). N-sector financing and investment \((I_t)\) plans are determined by the equilibria of the credit market game, characterized by the next proposition.

**Proposition 2.1 (Symmetric Non-diversion Credit Market Equilibria (CME))** Given prices, there is investment in the production of N-goods if and only if

\[
Re^{e^c}_{t+1} := \beta \theta \left[ ut+1 \frac{pt+1}{pt} \right] \geq \frac{1}{\delta} > \frac{h}{ut+1}
\]  

(12)

Suppose (12) holds. Then,

i In a financially repressed regime credit and investment are:

\[
b^n_t = [m^n - 1] w_t, \quad I_t = m^n \frac{w_t}{pt}, \quad \text{with } m^n = \frac{1}{1 - h\delta}. \tag{13}
\]

ii In a financially liberalized regime there is a ‘safe’ CME as in (13). If in addition \(ut+1 = u < 1\) and \(\frac{\beta \theta \theta^{u+1}}{pt} < \frac{h}{u}\), there also exists a ‘risky’ CME in which currency mismatch is optimal \((b^n_t = 0)\). Credit and investment are:

\[
b_t = [m^r - 1] w_t, \quad I_t = m^r \frac{w_t}{pt}, \quad \text{with } m^r = \frac{1}{1 - u - h\delta}. \tag{14}
\]

This proposition follows from the results in Schneider and Tornell (2004). To see the intuition notice that, given that all other entrepreneurs choose the safe plan (i), an entrepreneur knows that no bailout will be granted next period. Since lenders must break-even, the entrepreneur must internalize all bankruptcy costs. Thus, she will not set a diversion scheme and will hedge insolvency risk by denominating all debt in N-goods. Since the firm will never go bust and lenders must break even, the interest rate that the entrepreneur has to offer satisfies

\[1 + \rho^n_t = [1 + r] / Et(p_{t+1}).\]

Since (12) holds, investment yields a return which is higher than the opportunity cost of capital.\(^9\) Thus, the entrepreneur will borrow up to an amount that makes the credit constraint binding: \((1+r)b^n_t \leq h(w_t + b^n_t)\). Substituting this borrowing constraint in the budget constraint \(ptI_t = w_t + b^n_t\) generates the investment equation. Notice that a necessary condition for borrowing constraints to arise is \(h < 1 + r\). If \(h\), the index of contract enforceability, were greater than the cost of capital, it would always be cheaper to repay debt rather than to divert.

Given that all other entrepreneurs choose the risky plan (ii), a young entrepreneur expects a bailout in the low state, but not in the high state. The proposition shows that, in spite of the guarantees, diversion

\(^9\) The marginal return to investment is \(Et(p_{t+1})\Theta_t \beta k^{1-\beta}_t = \Theta_t \beta k^{1-\beta}_t - (\delta pt)^{-1} = Et(p_{t+1})\beta \beta - (\delta pt)^{-1}.\) This is because in an SE \(\Theta_t = \theta k^{1-\beta}_t, k_t = k_t\) and \(l_t = 1.\)
schemes are not optimal. Thus, borrowing constraints bind. Will the entrepreneur choose T-debt or N-debt? She knows that all other firms will go bust in the bad state (i.e., \( \pi(\underline{L}_{t+1}) < 0 \)) provided there is insolvency risk — i.e., \( \frac{\partial \rho_t}{\partial \rho_t} < \frac{1}{u} \). However, since there are systemic guarantees, lenders will get repaid in full. Thus, the interest rate on T-debt that allows lenders to break-even satisfies

\[
1 + \rho_t^{risky} = 1 + r
\]

It follows that the benefits of a risky plan derive from the fact that choosing T-debt over N-debt reduces the cost of capital from \( 1 + r \) to \( [1 + r/u] \). Lower expected debt repayments ease the borrowing constraint as lenders will lend up to an amount that equates \( u[1 + r]b_t \) to \( h[w_t + b_t] \). Thus, investment is higher relative to a plan financed with N-debt. The downside of a risky plan is that it entails a probability \( 1 - u \) of insolvency. Will the two benefits of issuing T-debt —more and cheaper funding— be large enough to compensate for the cost of bankruptcy in the bad state? If there is sufficient real exchange rate variability and \( u \) is not too low, expected profits under a risky plan exceed those under a safe plan: \( u \pi^r(\overline{p}_{t+1}) > u \pi^s(\overline{p}_{t+1}) + (1-u)\pi^s(\underline{p}_{t+1}) \).

To sum up, Proposition 2.1 makes three key points regarding the transition equations. If a risky plan is solvent, the young entrepreneur’s wage equals the marginal product of her labor, while under insolvency she just obtains a share \( \mu_w \) of revenues. Thus, in any SE the young entrepreneur’s cash flow is

\[
w_t = \begin{cases} 
[1 - \beta]p_t q_t & \text{if } \pi(p_t) \geq 0 \\
\mu_w m_t q_t & \text{if } \pi(p_t) < 0, 
\end{cases}
\]

(15)

Suppose for a moment that (12) holds, so that it is optimal to invest all funds in the production of N-goods: \( p_t I_t = m_t w_t \). It then follows from (15) that N-sector investment is

\[
I_t = \phi_t q_t,
\]

(16)

\[
\phi_t = \begin{cases} 
[1 - \beta]m_t & \text{if } \pi(p_t) \geq 0 \\
\mu_w m_t & \text{if } \pi(p_t) < 0, 
\end{cases}
\]

Since in an SE \( q_t = \theta I_{t-1} \), it follows from (10), (16) and the market clearing condition \( (d_t + I_t = q_t) \) that equilibrium N-output, prices and T-output evolve according to

\[
q_t = \theta \phi_{t-1} q_{t-1}
\]

(17)

\[
p_t = \alpha [q_t(1 - \phi_t)]^{\alpha-1}
\]

(18)

\[
y_t = [q_t(1 - \phi_t)]^\alpha = \frac{1 - \phi_t}{\alpha} p_t q_t
\]

(19)
Clearly, for prices to be positive it is necessary that the share of N-output purchased by the N-sector $\phi_t$ is less than one:

$$h < u_{t+1}/\bar{p}$$  \hspace{1cm} (20)

Equations (16)-(19) form an SE provided the implied returns validate the agents’ expectations (specified in Proposition 2.1). The next two propositions characterize two such SE: a safe one in which crises never occur, and a risky one where all firms become insolvent in the low price state and are solvent in the high price state.

**Proposition 2.2 (Safe Symmetric Equilibria (SSE))**  There exists an SSE if and only if the degree of contract enforceability $h$ is low enough and N-sector productivity $\theta$ is large enough. In an SSE there is no currency mismatch ($b_t = 0$) and crises never occur ($u_{t+1} = 1$). Thus, the N-sector investment share is

$$\phi^s = \frac{1 - \beta}{1 - h\delta}; \quad \delta = \frac{1}{1+r}.$$

(21)

This proposition states that an SSE exists provided enforceability problems are severe, so that (i) there are borrowing constraints and (ii) $\phi_t < 1$; and productivity is high enough, so that the return on investment is attractive enough.

In an SSE all entrepreneurs select the safe plan of Proposition 2.1 during every period. This implies that there is no currency mismatch in the aggregate, and self-fulfilling crises are not possible ($u_{t+1} = 1$). Therefore, the production of N-goods has a positive net present value (i.e., (12) holds) if and only if $\beta \theta_{p+1} = \beta \theta^a (\phi^s)^{\alpha - 1} - \delta^{-1}$. This condition, as well as (20), hold provided $h$ is low enough and $\theta$ is high enough.

Next, we characterize Risky Symmetric Equilibria (RSE). We have seen that entrepreneurs will take on T-debt only if there is enough anticipated real exchange rate variability to generate high returns in the good state and a critical mass of insolvencies in the bad state. We now reverse the question and ask instead when a risky debt structure implies enough real exchange rate variability. That is:

(i) will there be a sufficiently high return in the good state to ensure that the ex-ante expected return is high enough ($R^g_t \geq 1 + r$)?

The following proposition provides answers to these questions, and it establishes that the self-reinforcing mechanism we described above is at work. On the one hand, expected real exchange rate variability makes it optimal for entrepreneurs to denominate debt in T-goods and run the risk of going bust. On the other hand, the resulting currency mismatch at the aggregate level makes the real exchange rate variable, validating agents’ expectations.

**Proposition 2.3 (Risky Symmetric Equilibrium (RSE))**  There exists an RSE if and only if the probability of crisis $(1 - u)$ is small enough, N-sector productivity $(\theta)$ is large enough, and the degree of contract enforceability $(h)$ is low, but not too low.

1. In any RSE multiple crises can occur during which all N-sector firms default and there is a sharp real depreciation. However, two crises cannot occur in consecutive periods.

2. In the RSE where there is a reversion back to a risky path in the period immediately after the crisis, all firms choose risky plans in no-crisis times and safe plans in crisis times. The probability of a crisis
and the N-sector’s investment share satisfy:

\[ 1 - u_{t+1} = \begin{cases} 1 - u & \text{if } t \neq \tau_i \\ 0 & \text{if } t = \tau_i \end{cases} \]

\[ \phi_t = \begin{cases} \phi_l & \text{if } t \neq \tau_i \\ \phi^c & \text{if } t = \tau_i \end{cases} \]

where \( \tau_i \) denotes a crisis time.

3 Growth

Here, we compare the long-run growth rates of credit and GDP along the financially repressed and liberalized regimes—characterized in Propositions 2.2 and 2.3. In Section 4.4 we consider the laissez-faire regime.

It follows from Proposition 2.1 that in an SE the credit extended to the N-sector, expressed in terms of N-goods, is given by

\[ B_t = \begin{cases} [\phi_t - (1 - \beta)]q_t & \text{if } \pi(p_t) \geq 0 \\ [\phi_t - \mu_w]q_t & \text{if } \pi(p_t) < 0 \end{cases} \]
Figure 1: Non Tradables Market Equilibrium

\[ q_i^S(p_i) = \alpha \phi_{i-1} q_{i-1} \]

\[ q_i^B(p_i) = \left( \frac{\alpha}{p_i} \right)^{\frac{1}{\gamma}} - \frac{1}{1-\phi^i} \]  
(N-Firms are Solvent)

\[ q_i^N(p_i) = \left( \frac{\alpha}{p_i} \right)^{\frac{1}{\gamma}} - \frac{1}{1-\phi^i} \]  
(N-Firms are Bankrupt)
Since N-goods are intermediate inputs, while T-goods are final consumption goods, gross domestic product equals the value of N-sector investment plus T-output: \( gdpt = p_I t + y_t \). It then follows from (16)-(19) that, in any SE, GDP is given by

\[
gdp_t = p_t \phi_t q_t + y_t = q_t^\alpha Z(\phi_t) = y_t \frac{Z(\phi_t)}{1 - \phi_t}, \quad Z(\phi_t) = \frac{1 - (1 - \alpha)\phi_t}{1 - \phi_t}^{1 - \alpha} \tag{24}
\]

As we can see, the key determinant of the evolution of GDP is the share of N-output commanded by the N-sector for investment: \( \phi_t \). This share is determined by the cash flow of young entrepreneurs and by the credit they can obtain.

### 3.1 Growth in a Financially Repressed Economy

In an SSE the investment share \( \phi_t \) is constant and equal to \( \phi^s \). Thus, (24) implies that GDP and T-output grow at the same rate.

\[
1 + \gamma^s := \frac{gdpt}{gdpt_{-1}} = \frac{y_t}{y_{t-1}} = \left( \theta \frac{1 - \beta}{1 - h\delta} \right)^\alpha = \left( \theta \phi^s \right)^\alpha \tag{25}
\]

Absent exogenous technological progress in the T-sector, the endogenous growth of the N-sector is the force driving growth in both sectors. As the N-sector expands, N-goods become more abundant and cheaper allowing the T-sector to expand production. This expansion is possible if and only if N-sector productivity (\( \theta \)) and the N-investment share (\( \phi^s \)) are high enough, so that credit and N-output can grow over time:

\[
\frac{B_t}{B_{t-1}} = \frac{\phi_t}{\phi_{t-1}} = \theta \phi^s > 1.
\]

Notice that for any positive growth rate of N-output, \( \gamma^s \) increases with the intensity of the N-input in the production of T-goods (\( \alpha \)).

### 3.2 Growth in a Financially Liberalized Economy

Proposition 2.3 shows that any RSE is composed of a succession of lucky paths punctuated by crisis episodes. In the RSE characterized by (2.3) the economy is on a lucky path at time \( t \) if there has not been a crisis either at \( t - 1 \) or at \( t \). Since along a lucky path the investment share equals \( \phi^l \), (24) implies that the common growth rate of GDP and T-output is

\[
1 + \gamma^l := \frac{gdpt}{gdpt_{-1}} = \frac{y_t}{y_{t-1}} = \left( \theta \frac{1 - \beta}{1 - h\delta + u} \right)^\alpha = \left( \theta \phi^l \right)^\alpha \tag{26}
\]

A comparison of (25) and (26) reveals that as long as a crisis does not occur, growth in a risky economy is higher than in a safe economy. Along the lucky path the N-sector undertakes insolvency risk by issuing T-debt. Since there are systemic guarantees, financing costs fall and borrowing constraints are relaxed, relative to a safe economy. This increases the N-sector’s investment share (\( \phi^l > \phi^s \)). Since there are sectorial linkages (\( \alpha > 0 \)), this increase in the N-sector’s investment share benefits both the T- and the N-sectors and fosters faster GDP growth.

\[\text{10 The mechanism by which higher growth in the N-sector induces higher growth in the T-sector is the decline in the relative price of N-goods that takes place in a growing economy } \frac{p_{t+1}}{p_t} = [\theta \phi^s]^{\alpha - 1}. \text{ If there were technological progress in the T-sector, there would be a Balassa-Samuelson effect and the real exchange rate would appreciate over time. To see this, suppose the technological parameter in the T-production function grows over time } \frac{a_{t+1}}{a_t} = (1 + g). \text{ Then price dynamics are given by } \frac{p_{t+1}}{p_t} = (1 + g)[\theta \phi^s]^{\alpha - 1}.\]
However, in a risky economy a self-fulfilling crisis can occur with probability $1 - u$, and during a crisis episode growth is lower than along a safe path. We have seen that any crisis episode consists of at least two periods: in the first period the financial position of the N-sector is severely weakened and the investment share falls from $\phi^l$ to $\phi^c < \phi^s$; then in the second period it jumps back to $\phi^l$. Since these transitions occur with certainty, the mean crisis growth rate is given by:

$$1 + \gamma^{cr} = \left( \frac{\theta \phi^l}{\phi^s} \right)^{1/2} \left( \frac{\theta \phi^c}{\phi^l} \right)^{1/2} = \left( \theta (\phi^l / \phi^c)^{1/2} \right)^{\alpha}$$

The second equality in (27) shows that the average loss in GDP growth stems only from the fall in the N-sector’s average investment share: $(\phi^l/\phi^c)^{1/2}$. This reduction comes about through two channels: financial distress (indexed by $\mu^{\phi - \beta}$) and a reduction in risk taking and leverage (indexed by $1 - h \delta (1 - h u)$. Notice that variations in GDP growth generated by real exchange rate changes at $\tau$ and $\tau + 1$ cancel out. Appendix A analyzes the costs of crises.

A crisis has long-run effects because N-investment is the source of endogenous growth, and so the level of GDP falls permanently. This raises two questions: is mean long-run GDP growth in a risky economy greater than in a safe one? Does an increase in risk taking (i.e., an increase in the probability of crisis) in a risky economy increase mean long-run GDP growth? The answers to these questions are not straightforward because an increase in the probability of crisis $(1 - u)$ has opposing effects on long-run growth. One the one hand, a greater $1 - u$ increases investment and growth along the lucky path by increasing the subsidy implicit in the guarantee and allowing firms to be more leveraged. On the other hand, a greater $1 - u$ makes crises more frequent. Therefore, to give a precise answer to the questions we have raised, we compute the limit distribution of GDP’s growth rate.

**Growth Limit Distribution.** Next, we derive the limit distribution of GDP’s compounded growth rate $(\log(gdp) - \log(gdp_{t-1}))$ along the RSE characterized in Proposition 2.3. In this RSE firms undertake credit risk the period after the crisis. In subsection 3.2.1 we consider alternative RSEs where a crisis is followed by a cool-off phase during which safe plans are undertaken.

Recall that in any RSE two crises cannot occur in consecutive periods. It follows from (22), (26) and (27) that the growth process follows a three-state Markov chain characterized by

$$\Gamma = \left( \begin{array}{c}
\log \left( \frac{\theta \phi^l}{\phi^s} \right) \\
\log \left( \frac{\theta \phi^c}{\phi^l} \right) \\
\log \left( \frac{\theta \phi^s}{\phi^c} \right)
\end{array} \right), \quad T = \left( \begin{array}{ccc}
u & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0
\end{array} \right)$$

The three elements of $\Gamma$ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element $T_{ij}$ of the transition matrix is the transition probability from state $i$ to state $j$. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T \Pi = \Pi$. Thus, $\Pi = \left( \frac{u}{u + 1 - u}, \frac{1 - u}{u + 1 - u}, \frac{1 - u}{u + 1 - u} \right)$, where the elements of $\Pi$ are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is
$E(1 + \gamma^r) = \exp(\mu_1)$. That is,

$$E(1 + \gamma^r) = (1 + \gamma^i)\mu_0(1 + \gamma^{ce})^{1-\omega} = \theta^s(\phi^l)^\omega(\phi^f^l \phi^c)^\frac{1-\omega}{\omega}, \text{ where } \omega = \frac{u}{2-u} \quad (28)$$

A comparison of long run GDP growth rates in (25) and (28) reveals the trade-offs involved in following safe and risky growth paths, and allows us to determine the conditions under which credit risk is growth enhancing. Rearranging (28), we derive in the following proposition.

**Proposition 3.1 (Long-run GDP Growth)** In a RSE the mean long-run GDP growth rate is given by

$$E(1 + \gamma^r) = (1 + \gamma^s)^\alpha \left( \frac{\phi^l}{\phi^s} \right)^{\frac{1}{\alpha}} \left( \frac{\mu_w}{1-\beta} \right)^{\frac{1}{\alpha} - u} \quad (29)$$

1. There is risky equilibrium such that mean long-run GDP growth is greater than in a safe equilibrium only if financial distress during crises is not too severe (i.e., $\bar{t}^d = 1 - \frac{\mu_w}{1-\beta} < \bar{t}^d$).

2. If $\bar{t}^d < \bar{t}^d$, there exists an $h^* < u\beta\delta^{-1}$, such that mean growth is greater in a risky than in a safe equilibrium if and only if the degree of contract enforceability satisfies $h > h^*$:

$$h^* = \frac{1 - (1 - \bar{t}^d)^{1-u}}{u - (1 - \bar{t}^d)^{1-u} - \frac{1}{\delta}} \quad \bar{t}^d = 1 - (1 - \beta^{-1})^{\frac{1}{\alpha}}. \quad (30)$$

The proposition establishes two conditions for risk-taking to be growth enhancing. First, financial distress costs, as measured by the fall in a firm’s internal funds from $1 - \beta$ to $\mu_w$, cannot be excessively large to allow for risk-taking to increase long run growth. Second, the degree of contract enforceability $h$ needs to be high enough so that the leverage effect associated with risk-taking is sufficiently strong.

Rewriting $h > h^*$ as $(1 - u) [\log(1 - \beta) - \log(\mu_w)] < \log(\phi^l) - \log(\phi^s)$ makes clear what are the costs and benefits associated with a risky path. A risky economy outperforms a safe one if the benefits of higher investment in no-crisis times ($\phi^l > \phi^s$) compensate for the shortfall in internal funds and investment in crisis times ($\mu_w < 1 - \beta$) weighted by the frequency of crisis $(1 - u)$.

Notice that an increase in distress costs can be compensated by an increase in the degree of contract enforceability. The latter increases leverage and amplifies the benefits of risk-taking ($\partial \phi^f / \partial h > \partial \phi^c / \partial h$). However, as $h$ is bounded above to ensure the existence of an RSE ($\phi^l < 1 \iff h < u\beta\delta^{-1}$), an increase in contract enforceability can compensate for large but not arbitrarily large financial distress costs (i.e., $\mu_w \rightarrow 0$).

Figure 2 exhibits one realization of the paths of GDP, credit, T- and N-output associated with a set of parameters satisfying the conditions in Propositions 2.2 and 2.3. This figure makes clear that greater long run growth comes at the cost of (rare) crises. Notice that since N-goods are used as inputs in both sectors, higher N-sector investment leads to a lower initial level of T-output in a risky economy ($y_0 = \left[ q_0 (1 - \phi^l) \right]^{\alpha} < [q_0 (1 - \phi^s)]^{\alpha} = y_0^s$). Over time, however, T-output along the risky path will overtake that in a safe path.

---

1. $E(1 + \gamma^r)$ is the geometric mean of $1 + \gamma^i, 1 + \gamma^{ce}$ and $1 + \gamma^{cl}$.
2. How large can “not too large” be?

<table>
<thead>
<tr>
<th>$1 - \beta = 0.2$</th>
<th>$1 - \beta = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.85</td>
</tr>
<tr>
<td>$l_0$</td>
<td>95.4%</td>
</tr>
<tr>
<td>$\bar{t}^d$</td>
<td>74.2%</td>
</tr>
</tbody>
</table>
Figure 2: Risky vs Safe Economy

parameters: $\theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad I^d = 70\% \quad 1 - u = 5\%$
Figure 3: Limit Distribution of GDP

Figure 3 illustrates the limit distribution of GDP growth rates by plotting different GDP paths corresponding to different realizations of the sunspot process. Most of the risky paths outperform the safe path, except for a few unlucky risky paths. If we increased the number of paths, the cross section distribution would converge to the limit distribution.

Figure 4 exhibits the two effects of an increase in the probability of crisis \((1 - u)\). A reduction in \(u\) increases the investment multiplier \(m^r\) at a point in time, but it also increases the frequency of crises. The figure shows that for high \(u\) the first effect dominates and the long-run mean growth rate of GDP goes up. Importantly, \(u\) cannot be reduced indefinitely. After a certain point an RSE ceases to exist.

Finally, Figure 5 shows risky growth paths associated with different degrees of crisis’ financial distress. As we can see, even if 90% of N-sector cash flow is lost during a crisis, a risky economy can outperform a safe economy over the long run.

\[\text{parameters} \quad \theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l'^s = 70\% \quad 1 - u = 5\%\]
Figure 4: GDP Growth and Credit Risk

\[
\begin{align*}
\text{log(GDP)} & = -\theta + \alpha \log(\text{time}) + \beta \log(\text{time})^2 + \ln(\text{logarithm of parameters}) \\
\text{parameters} & : \theta = 1.65 \quad \alpha = 0.35 \quad \beta = 0.76 \quad 1 - \beta = 0.2 \quad t^4 = 70 \%
\end{align*}
\]
Figure 5: GDP Growth and Financial Distress Costs \( t^d = 1 - \frac{\theta}{1 - \beta} \)

\[ \text{log(GDP)} \]

parameters: \( \theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad 1 - \alpha = 5\% \)
3.2.1 Post-Crisis Cool-Off Phase and Growth

Here, we show that from the perspective of long-run growth, nothing is gained by delaying the onset of the new risky phase.

In Proposition 2.3, we characterized a RSE where there is a reversion back to a risky path in the period immediately after the crisis. We then compared growth in such a risky economy—where risk-taking occurs whenever it is possible—to growth in a safe economy where risk-taking never occurs. The comparison of these polar cases makes the argument transparent, but opens the question of whether the growth results presented in Proposition 3.1 are applicable to recent experiences in which systemic crises have been followed by protracted periods of low leverage, low investment and low growth.\textsuperscript{13} In order to address this issue, we construct an alternative RSE under which a crisis is followed by a cool-off phase during which all agents choose safe plans. The cool-off phase can be interpreted either as a period in which agents believe that others are following safe strategies or as a period during which agents are prevented from taking on risk.\textsuperscript{14}

To keep the model tractable, we assume that in the aftermath of a crisis, all agents follow safe plans with probability $\zeta$. Hence, a crisis is followed by a cool-off phase of average length $1/(1 - \zeta)$ before there is reversion to a risky path.$^{15}$ We show in the appendix that in this case, the mean long-run GDP growth rate is

$$E(1 + \gamma^c) = (\theta \phi^s)^{\alpha} \left( \frac{\phi^d}{\phi^s} \right)^{\frac{1 - \zeta}{\zeta}(1 - \eta)} \left( \frac{\mu_w}{1 - \beta} \right) \frac{1}{(1 - \zeta)^{1 - \mu_w(1 - \zeta)}},$$

which generalizes the growth rate of Proposition 3.1. Comparing (31) with (25) we can prove the following Lemma.

\textbf{Lemma 3.1} Consider an RSE where a crisis is followed by a cool-off period of average length $1/(1 - \zeta)$. Then:

1. The conditions under which mean long-run GDP growth is greater in a risky than in a safe equilibrium are independent of $\zeta$, and are the same as those in Proposition 3.1.

2. The shorter the average cool-off period $1/(1 - \zeta)$, the higher the mean long-run GDP growth in a RSE.

The reason why the growth-enhancing properties of risk taking—stated in Proposition 3.1—are independent of $\zeta$ is that during the cool-off phase the economy grows at the same rate as in a safe equilibrium. Part 2 makes the important point that the faster risk-taking resumes in the wake of crisis, the higher will be mean long-run growth.$^{13}$ Figure 1 is suggestive of such reversion to a safe path in Thailand after the 1997 crisis.$^{14}$ Or alternatively as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.$^{15}$ The average length of the cooling off period is computed as:

$$\lambda = (1 - \zeta) \sum_{k=0}^{\infty} \xi^{k-1} k = \frac{1}{1 - \xi}$$

\textsuperscript{13}Figure 1 is suggestive of such reversion to a safe path in Thailand after the 1997 crisis.

\textsuperscript{14}Or alternatively as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.

\textsuperscript{15}The average length of the cooling off period is computed as:
4 Production Efficiency and Fiscal Implementability

We have considered an endogenous growth model where the financially constrained N-sector is the engine of growth because it produces the intermediate input used throughout the economy. Thus, the share of N-output invested in the N-sector, \( \phi_t \), is the key determinant of economic growth. When \( \phi_t \) is too small T-output is high in the short-run, but long-run growth is slow. In contrast, when \( \phi_t \) is too high, there is inefficient accumulation of N-goods. In this section we ask three questions. First, what is the Pareto optimal N-investment share sequence \( \{ \phi_t \} \)? Second, can this Pareto optimal investment sequence be replicated in a financially repressed economy? If not, can the present value of consumption be higher in a financially liberalized economy where agents undertake credit risk and crises occur? Third, is such an efficiency improving reallocation implementable? In particular, will T-sector agents be willing to foot the bill to finance the bailout guarantees associated with a risky economy? In Section 4.4 we consider the laissez-faire regime.

4.1 Pareto Optimality

In our set-up, N-goods are intermediate inputs, while T-goods are final consumption goods. Consider then a central planner who maximizes social welfare by investing the supply of N-goods in the T-sector (\( [1 - \phi_t] q_t := d_t \)) and the N-sector (\( \phi_t q_t \)), as well as by assigning sequences of T-goods to consumers and entrepreneurs for their consumption:

\[
\max_{\{c_t, c^e_t, \phi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \left[ (1 - \nu) u(c_t) + \nu c^e_t \right], \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \delta^t [c_t + c^e_t - y_t] \leq 0
\]

\[
y_t = [1 - \phi_t]^\alpha q^\alpha_t, \quad q_{t+1} = \theta \phi_t q_t
\]

Clearly, Pareto optimality implies efficient accumulation of N-inputs: because optimal consumption is a function of the present value of income, the planner should choose the investment sequence \( \{ \phi_t \} \) to maximize the present value of T-production: \( \sum_{t=0}^{\infty} \delta^t y_t \). We show in the Appendix that the Pareto optimal N-investment share is constant and equal to

\[
\phi^{po} = (\theta^\alpha \delta)^{1/\alpha - 1}, \quad \text{if} \quad \alpha < \log(\delta^{-1})/\log(\theta)
\]

The Pareto optimal share equalizes the discount rate \( \delta^{-1} \) to the intertemporal rate of transformation. A marginal increase in the N-sector investment share \( \partial \phi \) reduces today’s T-output by \( \alpha [(1 - \phi) q_t]^{\alpha-1} \partial \phi \), but increases tomorrow’s N-output by \( \theta \partial \phi \) and tomorrow’s T-output by \( \alpha [(1 - \phi) \theta \phi q_t]^{\alpha-1} \theta \partial \phi \). Thus, at an optimum \( \theta^\alpha \phi^{\alpha-1} = \delta^{-1} \).

Can a decentralized economy replicate the Pareto optimal allocation? The optimal N-investment share is determined by investment opportunities: \( \theta^\alpha \delta \). In contrast, in a decentralized safe economy the N-investment share \( \phi^s = \frac{1 - \beta}{1 + \beta} \) is determined by the credit market imperfections: the degree of contract enforceability \( h \) and the constrained sector’s cash flow \( 1 - \beta \). Clearly, if either \( h \) or \( 1 - \beta \) are low, the N-sector investment share will be lower than the Pareto optimal share: \( \phi^s < \phi^{po} \). That is, when the N-sector is severely credit constrained, low N-sector investment will keep the economy below production efficiency. For future reference we summarize with the following Proposition.
Proposition 4.1 (Bottleneck) \textit{N-sector investment in a safe economy is below the Pareto optimal level (i.e., there is a ‘bottleneck’) if there is low contract enforceability:}

\[ h < (1 - (1 - \beta)\theta (\theta \delta)^{-1})/\delta. \]

When there is a bottleneck, the share of N-inputs allocated to T-production should be reduced and that allocated to N-production should be increased in order to bring the allocation nearer to the Pareto optimal level. This reallocation reduces the initial level of T-output, but increase its growth rate and the present value of cumulative T-production. Can financial liberalization and the resulting adoption of credit risk induce this reallocation and bring the economy nearer to the Pareto optimum? Is there a sense in which social welfare increases? Recall that along a lucky path of an RSE the investment share is greater than the share in a safe economy. However, credit risk through currency mismatch makes the economy vulnerable to crises, which entail deadweight losses for the economy. In the next subsection, we consider the effects of crises and ask whether ex-ante the present value of consumption in a risky economy is greater than in a safe economy.

4.2 Present Value of Consumption

The expected discounted sum of consumers and entrepreneurs’ consumption is

\[ W = E_0 \left( \sum_{t=0}^{\infty} \delta^t (c_t + c_t') \right) = E_0 \left( \sum_{t=0}^{\infty} \delta^t [(1 - \alpha) y_t + \pi_t - T_t] \right) \]  

To derive the second equation in (34) notice that in equilibrium consumers’ income is \([1 - \alpha]y_t\), entrepreneurs’ income is equal to their profits \(\pi_t\), and the fiscal cost of bailouts is financed with lump-sum taxes \(T_t\).

In order to obtain a closed-form solution notice that at any \(t \geq 1\) profits equal the old entrepreneurs share in revenues minus debt repayments: \(\pi_t = \beta p_t q_t - L_t = \frac{\alpha}{1 - \delta} \beta y_t - \frac{\alpha}{1 - \delta} h \phi^s y_{t-1} \). Meanwhile, since at \(t = 0\) there is no debt burden, \(\pi_0 = \frac{\alpha}{1 - \delta} \beta y_0\). In a safe economy firms are always solvent and crises never occur. Thus, there are no bailouts and no taxes. It then follows from (34) that the present value of consumption equals the present value of T-output

\[ W^s = \sum_{t=0}^{\infty} \delta^t y_t^s = \frac{1}{1 - \delta (\theta \phi^s)^\alpha} y_0^s = \frac{(1 - \phi^s)^\alpha}{1 - \delta (\theta \phi^s)^\alpha} q_0^s \quad \text{if } \delta (\theta \phi^s)^\alpha < 1 \]  

Consider a risky economy. Along the lucky path, the investment share is greater than in a safe economy. Thus, if there is a bottleneck and crises are rare events, the present value of T-output along the lucky path is greater than in a safe path. However, along a lucky path a crisis can occur with probability \(1 - u\). The question then arises as to whether it is worthwhile to incur the crisis costs in order to attain higher T-output growth.

A crisis involves three costs. First, there is a fiscal cost. Lenders receive a bailout payment equal to the debt repayment they were promised: \(L_t = u^{-1} h \phi^l p_t q_{t-1} \). Since the bailout agency recuperates only a share \(\mu \leq \beta\) of firms revenues \(p_t q_t\), while the rest is dissipated in bankruptcy procedures, the fiscal cost of a

\[ \text{In our model economy, the government and consumers have access to complete financial markets and their discount rate equals the riskless interest rate, so their consumption is constant over time. Furthermore, N-sector entrepreneurs are risk-neutral. Thus, we could identify the expected discounted sum of consumers and entrepreneurs’ consumption with ex-ante social welfare.} \]
The value of consumption in a risky economy can be greater than in a safe economy. During crisis borrowing constraints are tighter than in a safe economy because an N-firm’s net worth is \( \mu_w p_T q_T \) instead of \( [1 - \beta] p_T q_T \) and risk taking is curtailed: only safe plans are financed. Finally, since during a crisis all N-firms go bust, old entrepreneurs’ profits are zero.

The deadweight loss of a crisis for the economy as a whole is lower than the sum of these three costs. During a crisis there is a sharp redistribution from the N- to the T-sector generated by a severe real depreciation (a firesale). Thus, some of the costs incurred in the N-sector show up as greater T-output and consumers’ income. We show in the Appendix that after netting out the costs and redistributions, a crisis involves two deadweight losses: (i) the revenues dissipated in bankruptcy procedures: \( [\beta - \mu] p_T q_T \); and (ii) the fall in N-sector investment due to its weakened financial position: \( [(1 - \beta) - \mu_w] p_T q_T \). Using the market clearing condition \( \phi_T = [1 - \phi_t] p_T q_T \), we have that the sum of these two deadweight losses equals \( \frac{\alpha}{1 - \beta} [1 - \mu - \mu_w] y_T \) in terms of T-goods. Thus, in an RSE the present value of consumption is given by

\[
W^r = E_0 \sum_{t=0}^{\infty} \delta^t k_t y_t, \quad k_t = \begin{cases} 
    1 - \frac{\phi^f(1 - \mu - \mu_w)}{1 - \phi^s} & \text{if } t = \tau_i \\
    1 & \text{otherwise},
\end{cases}
\]

where \( \tau_i \) is a crisis time. In order to compute this expectation we need to calculate the limit distribution of \( k_t y_t \). We do this in the Appendix and show that it is equal to

\[
W^r = \frac{1 + \delta(1 - u) \left[ \theta \phi^f \frac{1 - \phi^s}{1 - \phi^f} \right]^\alpha k^c}{1 - \left[ \theta \phi^f \right]^\alpha \delta u - \left[ \theta^2 \phi^f \phi^c \right]^\alpha \delta^2 (1 - u) [(1 - \phi^f) q_0]^\alpha} \tag{37}
\]

By comparing (35) and (37) we can determine the conditions under which the ex-ante present value of consumption is greater in a risky economy.

**Proposition 4.2** If crises are rare events and the costs of crises \{\( \beta/\mu \), \( (1 - \beta)/\mu_w \)\} are small, then ex-ante present value of consumption in a financially liberalized economy is greater than in a financially repressed economy if and only if there is a bottleneck (\( \phi^s < \phi^{po} \)).

If crises entail small bankruptcy costs (\( \mu \rightarrow \beta \)) and mild financial distress (\( \mu_w \rightarrow 1 - \beta \)), the only first order effect of a crisis is to reduce transitorily the N-sector’s investment share from \( \phi^f \) to what it would have been in a safe economy (\( \phi^s \)). Thus, in this limit case the investment share in the risky economy would never be lower than in the safe one. Hence, if there is a ‘bottleneck’ (\( \phi^s < \phi^{po} \)) and crises are rare events, the greater average investment share will increase the present value of T-output and hence the present value of consumption.

Small crisis costs are sufficient, but not necessary, for the result stated in Proposition 4.2. The present value of consumption in a risky economy can be greater than in a safe one even if the static costs of crisis (i.e., the share of output lost in bankruptcy) is large, as long as the dynamic costs—i.e., the financial distress costs—are not excessively large. Figure 6 shows the welfare differential between safe and risky economies \( (W^r - W^s) \) for different bankruptcy costs \( (l^b = 1 - \frac{q}{\beta}) \) and financial distress costs \( (l^d = 1 - \frac{\mu_w}{1 - \beta}) \). As we can see, the welfare gains can be positive even if 100% of revenues are dissipated in bankruptcy procedures \( (\mu \rightarrow 0) \). There can also be positive gains for severe financial distress costs \( (l^d = 80\%) \). However, they are
negative when \( l^d \rightarrow 100\% \). The reason for this asymmetry is that bankruptcy costs are a static loss, while financial distress costs have dynamic effects. In our endogenous growth set-up, the reduction in N-sector investment shifts the growth paths of both sectors downwards. Such an unrecoverable long term loss reduces the discounted sum of T-production over the whole post-crisis period.\(^{17}\) By contrast, the gains are almost insensitive to bankruptcy costs.

The gain associated with undertaking credit risk is increasing in the probability of crisis \((1 - u)\). This does not mean that this probability can be arbitrarily large. As we have discussed earlier, an RSE exists only if crises are rare events. In panel (a) of Figure 7, we show how \( W^r - W^s \) varies over a range of crisis probabilities between 0 and 8%. Except when the financial distress cost of crises is very high, the risky economy dominates the safe economy. This difference is amplified by a limited increase in credit risk. In contrast, if crisis costs are very large, \( W^r - W^s < 0 \) and any increase in risk reduces \( W^r \) further. Finally, panel (b) of Figure 7 shows that the gains, in terms of the present value of consumption, are increasing in the intensity of N-inputs in T-production \((\alpha)\). A greater \( \alpha \) strengthens the sectorial linkage and thus increases the benefits of relaxing the borrowing constraint in the N-sector.

\(^{17}\) A second order welfare cost of crises is the variability in the level of investment (shift from \( \phi^d \) to \( \phi^c \) and back). Recall that the Pareto optimal investment share is constant.
Figure 7: Production Efficiency and Credit Risk

a. For different levels of Financial Distress Costs

parameters : $\theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l^d = 70\%$

b: for different intensities of Non-Tradables Input in Tradable Production

parameters : $\theta = 1.65 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l^d = l^b = 70\%$
4.3 Fiscal Implementability

Proposition 4.2 has established that the present value of consumption can be greater in a risky economy even if bailout costs are funded domestically via lump-sum taxes. Systemic bailout guarantees are necessary to induce agents to undertake insolvency risk (through currency mismatch). We have seen that such a risky strategy eases borrowing constraints and leads to a greater mean growth of N-output even along a path where crises do occur. As a result, T-production will enjoy cheaper and more abundant N-inputs, and its growth rate will also increase. This benefits consumers because they receive a share $1 - \alpha$ of T-output as income.

But, is a bailout scheme implementable? Will consumers be willing to foot the bill? In particular, will consumers at date zero be willing to purchase an insurance that promises to cover any future bankruptcy costs associated with the guarantees? Since the representative consumer has access to complete capital markets, he can perfectly smooth the cost of the guarantees. His lifetime budget constraint is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t [c_t - (1 - \alpha)y_t + T_t] \leq 0,$$

where $T_t$ is the tax that will finance the bailouts. Since the consumer’s share in T-output is $1 - \alpha$, his ex-ante welfare in a financially repressed and a financially liberalized economies are, respectively

$$C^s = [1 - \alpha]W^s, \quad C^r = \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t (y_t[1 - \alpha] - T_t)$$

(38)

The consumer will be willing to finance the bailout if and only if $C^s > C^r$:

$$1 + \delta(1 - u) \left[ \theta \phi^{\alpha} \left( \frac{1 - \phi}{1 - \phi^\alpha} \right) \right] \left( K^T_c \right) \left( [1 - \phi^\alpha]q_0 \right)^\alpha > \left( (1 - \phi^s)q_0 \right)^\alpha,$$

(39)

where $K^T_c$ is defined in (56) in the Appendix. The funding of the guarantees by consumers operates a redistribution from the non-constrained T-sector to the constrained N-sector. If (39) holds, such a redistribution is to the mutual benefit of both sectors. It is a Pareto-improving policy. Figure 8 exhibits the consumer’s net welfare gain when he finances all the bailout costs for $1 - \alpha = 0.35\%$. By comparing Figures 7 and 8 we can see that when social welfare gains are present, consumers welfare gains are also present, but in a smaller proportion.

4.4 Laissez-Faire Regime

To be added.

5 Related Literature.

To be added.

6 Conclusions

We have considered a two-sector economy to analyze how financial liberalization can help improve the allocation of resources—by increasing leverage in constrained sectors—but at the same time it can generate new states-financial black holes-under which systemic insolvencies occur.
We have shown that in an economy with credit market imperfections, financial liberalization can help overcome obstacles to growth by easing financing constraints of bank-dependent sectors with easy access to neither stock markets nor international capital markets. However, as a side effect financial fragility arises and thus crises occur from time to time. In other words, the trade-off is not fragility versus no fragility. The trade-off is: fragility and growth versus no fragility and no growth.

We have established conditions under which the bankruptcy and financial distress costs of crises are outweighed by the benefits of higher growth. Furthermore, we have established conditions under which the financially unconstrained sector will find it profitable to fund the systematic bailout guarantees that support the risky credit path along which the constrained sector grows faster. Under this scheme the unconstrained sector can also grow faster because it faces less severe bottlenecks—i.e., more abundant inputs produced by the constrained sector.

Our results should provide a caution when interpreting the effects of financial liberalization. From the finding that liberalization has lead to more bumpiness, one should not conclude that liberalization per-se is bad either for growth or for welfare. Furthermore, policies intended to eliminate risk taking and fragility might have the unintended effect of blocking the forces that generate financial deepening and growth. At the same time, our analysis shows that the other extreme—a lack of financial regulation—might also be harmful.

\[ \text{parameters } : \theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l^b = 70\% \]
In an any-thing-goes regime where borrowers can issue option like securities, the presence of systemic bailout guarantees might lead to excessive leverage and a lack of discipline in lending decisions.

Appendix

A. The Costs of Crises

During a crisis there are widespread bankruptcies, which generate deadweight losses as well as sectorial redistributions. Here, we net out these crises costs and show that the growth costs of crisis reduce to the fall in the N-sector’s investment share, as expressed in (27).

18 Although the main objective of the model is to address long-run issues, it is reassuring that it can account for these key stylized facts of recent financial crises, a sharp real depreciation that coincides with a fall in credit growth, as well as the asymmetric sectoral response of N- and T-sectors.

19 This is because young entrepreneurs income is only \( \mu w p r q_t \) instead of \( [1 - \beta] p r q_t \), and at \( \tau \) entrepreneurs can only choose safe plans in which there is no currency mismatch (by Proposition 2.3).

20 To interpret (27) note that variations in the investment share \( \phi_t \) have lagged and contemporaneous effects on GDP. The lagged effect comes about because a change in \( \phi_t \) affects next period’s GDP via its effect on N-output: \( q_{t+1} = \theta I_t = \theta \phi_{t+1} q_t \).

Using (24) and \( y_t = (1 - \phi_t) q_t \), the contemporaneous effect can be decomposed as:

\[
\frac{\partial g_{dp}}{\partial \phi_t} = \frac{\alpha y_t}{1 - \phi_t} + p q_t + q_t \phi_t \frac{\partial p}{\partial \phi_t} = q_t \phi_t \frac{\partial p}{\partial \phi_t}
\]

The first two terms capture variations in T-output and N-investment, while the third reflects real exchange rate fluctuations. Market clearing in the N-goods market – i.e., \( (1 - \phi_t) p r q_t = \alpha y_t \) – implies that the induced changes in N-sector investment and T-output cancel out. Therefore, the contemporaneous changes in the investment share affect GDP contemporaneously only through its effect on the real exchange rate. Since \( GDP_t = Z(\phi_t) q_t^\alpha \), we can express \( q_t \phi_t \frac{\partial p}{\partial \phi_t} \) as \( q_t^\alpha \frac{\partial Z}{\partial \phi_t} \). Thus, we can interpret \( \frac{Z(\phi_t)}{Z(\phi_{t-1})} \) as the effect of real exchange rate fluctuations on GDP.

29
firesale prices and expand production. This leads to a sharp fall in the N-to-T output ratio in the wake of crisis. The deadweight losses derive from the financial distress and the bankruptcy costs generated by crises. The former leads to a contraction in N-investment and thus has a long-run effect on output. In contrast, bankruptcy costs have only a static fiscal impact, which is the cost of the bailout.

B. Proofs and Derivations

Proof of Proposition 2.2. In an SSE, during every period, all entrepreneurs choose the safe plan characterized in Proposition 2.1. Each entrepreneur will find it optimal to do so provided a majority of entrepreneurs chooses a safe plan and the marginal return to investment in the production of N-goods is no lower than 1 + \beta h_{t+1} := \frac{\beta \theta p_{t+1}}{p_t} \geq \delta^{-1}. Since in an SSE crises never occur, prices are deterministic: \( u_{t+1} = 1 \) and \( p_{t+1} = p_t \). Using (17) and (18) it follows that \( R_{t+1} = \beta \theta^\alpha (\phi^\alpha)^{\alpha - 1} \). Thus, an SSE exists if and only if \beta \theta^\alpha (\phi^\alpha)^{\alpha - 1} > \delta^{-1} and (20) holds. These two conditions are equivalent to

\[
h < \bar{h} = \beta \delta^{-1}, \quad \theta > \bar{\theta} = [\delta \beta (\phi^\alpha)^{\alpha - 1}]^{-1/\alpha}
\]  

(40)

Proof of Proposition 2.3. The proof is in two parts. In part A we consider the case in which two crises do not occur in consecutive periods. Then, in part B we show that two crises cannot occur in consecutive periods.

Part A. Consider an RSE in which all entrepreneurs choose the risky plan characterized in Proposition 2.1 during every period, except when a crisis erupts, in which case they choose safe plans. In a no-crisis period, given that all other entrepreneurs choose a risky plan, an entrepreneur will only if \( R_{t+1} := u \beta \theta \bar{p}_{t+1} \geq 1 + r \), and \( \pi(p_{t+1}) < 0 \). To determine whether these conditions hold note that in an RSE the investment share \( \phi_{t+1} \) equals \( \phi^c \) if N-firms are solvent, while \( \phi_{t+1} = \phi^f \) if they are insolvent. Replacing these expressions in the equations for cash flow (15), N-output (17) and prices (18), it follows that

\[
R_{t+1} \geq \frac{1}{\alpha} \iff u \bar{R}(u) + [1 - u] R(u) \geq \frac{1}{\alpha}, \quad \bar{R}(u) := \beta \theta^\alpha \left[ \frac{1}{\phi^c} \right]^{1 - \alpha}
\]  

(41)

\[
\pi(p_{t+1}) < 0 \iff \bar{R}(u) < \frac{1}{\bar{\theta}}, \quad \bar{R}(u) := \beta \theta^\alpha \left[ \frac{1}{\phi^c} \right]^{1 - \alpha} \left[ 1 - \phi^c \right]^{1 - \alpha}
\]  

(42)

To derive (42) we have used \( \pi(p_{t+1}) = \beta \bar{q}_{t+1} - L_{t+1} = \beta \alpha [1 - \phi^c]^{\alpha - 1} [\theta (\phi^c) q_t]^{\alpha} - u^{-1} h_{t+1} \alpha [1 - \phi^c]^{\alpha - 1} q_t^{\alpha} \). Consider next a crisis period. Given that all other entrepreneurs choose a safe plan, an entrepreneur will find it optimal to do so if and only if \( R_{t+1} := \beta \theta p_{t+1} / p_t \geq \delta^{-1} \). Since in the post-crisis period there can be no crisis, it follows from the proof of Proposition 2.2 that this condition is equivalent to \beta \theta^\alpha (\phi^\alpha)^{\alpha - 1} \geq \delta^{-1}.

Clearly, this condition is implied by (41). It follows that there exists an RSE where two crises do not occur in consecutive periods if and only if (41) and (42) hold and parameters satisfy (20), which is given by

\[
h \delta < u \bar{\theta}
\]  

(43)

"Only if." We prove that an RSE exists only if \( u > \frac{\alpha}{\beta} \), \( \theta > \bar{\theta} \), and \( h < \bar{h} < \bar{l} \) in three steps.

Step 1. For any \( \theta \in \mathbb{R}^+ \) and any \( h \in \mathbb{R}^+ \) there exists no RSE if \( u \to 0 \). To prove this, let \( u \to 0 \). Since \( \theta \) is bounded and \( 1 - \beta < \phi^c \to 1 \), it follows that \( \lim_{u \to 0} u \bar{R}(u) = 0 \). Therefore, (41)-(43) imply that when \( u \to 0 \) an RSE exists if and only if \( \frac{\alpha}{\beta} < \bar{\theta} \) and \( \frac{\alpha}{\beta} < \bar{R}(u) \to \frac{h}{\alpha} \), which is a contradiction.
Step 2. For any \( u \in (0, 1) \) and for any \( \theta \in \mathbb{R}^+ \) there exists no RSE if \( h > \overline{h} \) or \( h < \overline{h} \), where

\[
\overline{h} = \frac{\beta u}{\delta}, \quad h = \frac{1}{\delta} \left( \frac{1 - \phi^c}{1 - \phi} \right)^{1 - \alpha} \left( \frac{1}{u} - 1 \right)^{-1}, \quad 0 < h < \overline{h}
\]  

(44)

Notice that \( h < \overline{h} \) is equivalent to (43), and that (41) and (42) hold if and only if \( \delta^{-1} \left( 1 - \right) \leq \overline{h}(u) < \frac{1}{\delta} \left( \frac{1 - \phi^c}{1 - \phi} \right)^{1 - 1} \), which holds only if \( h > \overline{h} \).

Step 3. For any \( u \in (0, 1) \) and for any \( h \in \mathbb{R}^+ \) there exists no RSE if \( \theta < \bar{\theta} \), where

\[
\theta = \left( \frac{h}{u^\beta} \left[ \phi^i \right]^{1 - \alpha} \left[ 1 - \phi^c \right]^{1 - \alpha} \right)^{1/\alpha}
\]  

(45)

Notice that \( uR(u) + (1 - u)R(u) \) is decreasing in \( h \) and an RSE exist only if \( h > \overline{h} \). Thus, a necessary condition for an RSE to exist is \( uR(u) + (1 - u)R(u) \bigg|_{h=\overline{h}} > \delta^{-1} \), which is equivalent to (45).

“If.” To establish the existence of an RSE we show that when \( u \to 1 \) parameter restrictions (41), (42) and (43) are mutually consistent if \( (\theta, h) \in S \). Then, we replace \( \theta < \bar{\theta} \) by tighter bounds on \( h \).

Step 1. We show that for any \( \delta \in (0, 1) \), \( \alpha \in (0, 1) \), and \( \mu_w \in (0, 1 - \beta) \) an RSE exists if \( (\theta, h) \in S' = \{(\theta, h) \in R^2_+ \mid h < \overline{h}, \theta_n(h) < \theta < \theta_d(h)\} \). Let \( u = 1 \), for any \( \delta \in (0, 1) \) and \( \alpha \in (0, 1) \), (43) holds iff \( h < \overline{h} = \beta \delta^{-1} \) and (41) holds iff \( \theta \geq \theta_d(h) = \left[ \delta \beta \phi^c \alpha^{-1} \right]^{1/\alpha} \). Next, if \( u = 1 \), (42) becomes

\[
\frac{\left[ 1 - \phi^c \right]^{1 - \alpha}}{1 - \phi} < h \left[ \phi^c \right]^{1 - \alpha}
\]  

This condition holds for any \( \mu_w \in (0, 1 - \beta), h < \overline{h} \) and \( \theta > \theta_n(h) \) iff

\[
\theta < \theta_d(h) = \left[ \frac{1 - \phi^c}{1 - \phi} \right]^{1 - \alpha} \frac{h}{\beta} \quad \text{and} \quad h > \overline{h} = \frac{1}{\delta} \left[ \frac{1 - \phi^c}{1 - \phi} \right]^{1 - \alpha}
\]  

(46)

Notice that \( h > \overline{h} \) is necessary for \( \theta_n(h) < \theta_d(h) \) and that \( \overline{h} \) is unique. Furthermore, \( \theta_n(h) < \theta_d(h) \Leftrightarrow h - \frac{1}{\delta} \left[ \frac{1 - \phi^c}{1 - \phi} \right]^{1 - \alpha} > 0 \). This expression is strictly increasing in \( h \), it is satisfied if \( h \to \overline{h} \) and violated if \( h = 0 \). This ensures existence and unicity of a lower bound \( \overline{h} \).

Step 2. We show that the sets \( S' \) and \( S \) are equivalent. Consider the following three properties of \( \theta_n(h) \) and \( \theta_d(h) \) over \( (\overline{h}, \overline{h}) \), which are illustrated in the figure below: (i) \( \theta_n(h) < \theta_d(h) \); (ii) \( \theta_n(h) \) and \( \theta_d(h) \) are continuous and strictly increasing in \( h \); and (iii) \( \theta_n(h) = \theta_d(h) = \overline{h} \); \( \lim_{h \to \overline{h}} \theta_n(h) = \infty \) and \( \lim_{h \to \overline{h}} \theta_d(h) = (\beta \delta^{-1})^{-1/\alpha} \). It follows that for any \( (\theta, h) \in S' \), \( \theta > \overline{h} \) and \( h \in (h', h'') \), where \( h' = \theta_n^{-1}(\theta) \) and \( h'' = \min(\theta_n^{-1}(\theta), \overline{h}) \) where \( \theta^{-1}(\cdot) \) denotes the inverse function. Since \( \overline{h} < h' < h'' < \overline{h} \), we have that \( (\theta, h) \in S' \Rightarrow (\theta, h) \in S \). Similarly, for any \( (\theta, h) \in S \), \( h < \overline{h} \) and \( \theta_n(h) < \theta < \theta_d(h) \). Therefore, \( (\theta, h) \in S \Rightarrow (\theta, h) \in S' \).
Part B. We prove by contradiction that two crises cannot occur in consecutive periods. Suppose that if a crisis occurs at \( \tau \), firms choose risky plans at \( \tau \). We will show that it is not possible, under any circumstances, for firms to become insolvent in the low price state at \( \tau+1 \) (i.e., \( \pi(p_{\tau+1}) < 0 \)). It suffices to consider the case in which firms undertake safe plans at \( \tau+1 \), as \( \pi(p_{\tau+1}) \) is the lowest in this case. Along this path the N-investment share equals \( \phi = \tilde{\phi}_c := \mu_w m^r \) and \( \phi_{\tau+1} = \phi_c := \mu_w m^s \). Thus, \( \pi(p_{\tau+1}) = \beta \alpha(1 - \phi^c)^{\alpha-1}[\theta \phi^c q^c]^{\alpha} - u^{-1} h \alpha(1 - \phi^c)^{\alpha-1} q^c \), and

\[
\tilde{\pi}(p_{\tau+1}) < 0 \iff \beta \theta^\alpha \left[ \frac{1 - \phi^c}{1 - \phi^c} \right]^{1-\alpha} \frac{1}{\tilde{h}} < \frac{u}{u} \tag{47}
\]

Notice that the LHS of (41) is strictly lower than the LHS of (47) because: (i) \( \mu_w < 1 - \beta \), so \( \frac{1 - \phi^c}{\phi^c} > \frac{1 - \phi^c}{\phi^c} \); and (ii) \( \phi_i > \phi_e \). However, the RHS of (41) is strictly higher than the RHS of (47) because \( u > h \delta \) is necessary for an RSE to exist. This is a contradiction. □

Proof of Proposition 3.1. Growth Limit Distribution. In any RSE two crises cannot occur in consecutive periods. Here, we will derive the limit distribution of GDP’s compounded growth rate \( \log(gdp_t) - \log(gdp_{t-1}) \) along the RSE characterized in Proposition 2.3. In this RSE firms undertake credit risk the period after the crisis. It follows from (22), (26) and (27) that the growth process follows a three-state Markov chain characterized by

\[
\Gamma = \begin{pmatrix}
\log \left( (\theta \phi^c)^{\alpha} \right)
\log \left( (\theta \phi^c)^{\alpha} \frac{Z(\phi^c)}{Z(\phi^c)} \right)
\log \left( (\theta \phi^c)^{\alpha} \frac{Z(\phi^c)}{Z(\phi^c)} \right)
\end{pmatrix}, \quad T = \begin{pmatrix}
u & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0
\end{pmatrix} \tag{48}
\]

The three elements of \( \Gamma \) are the growth rates in the lucky, crisis and post-crisis states, respectively. The element \( T_{ij} \) of the transition matrix is the transition probability from state \( i \) to state \( j \). Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves \( T \Pi = \Pi \). Thus, \( \Pi = \left( \begin{array}{c}
u \frac{1 - u}{2 - u} \frac{1 - u}{2 - u} \frac{1 - u}{2 - u}
\end{array} \right) \), where the elements of \( \Pi \) are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is
$E(1 + \gamma^r) = \exp(\Pi\Gamma).$\(^{21}\)

We derive first the limit distribution of the growth rate process $\Delta \log(gdp_t) := \log(gdp_t) - \log(gdp_{t-1})$. Since in an RSE crises cannot occur in two consecutive periods, $\Delta \log(gdp_t)$ follows a three-state Markov chain characterized by the following growth vector and transition matrix

$$\Gamma = \begin{pmatrix}
\log(\left(\theta p\right)^{\alpha}) \\
\log(\left(\theta p\right)^{\alpha} Z(p^e)) \\
\log(\left(\theta p\right)^{\alpha} Z(p^f))
\end{pmatrix}, \quad T = \begin{pmatrix}
u & 1 - \nu & 0 \\
0 & 0 & 1 \\
\nu & 1 - \nu & 0
\end{pmatrix}$$

Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T^\prime \Pi = \Pi$. Thus, $\Pi'\ell = \begin{pmatrix} \frac{\nu}{2 - \nu}, \frac{1 - \nu}{2 - \nu}, \frac{1 - \nu}{2 - \nu} \end{pmatrix}$ and the geometric mean long run GDP growth rate—equation (28) in the text—is $E(1 + \gamma^r) = \exp(\Pi\Gamma)$. It then follows from (25) and (28) that

$$\gamma^r > \gamma^s \iff \left(\frac{\mu_\tau}{1 - \beta}\right)^{1 - u} > \frac{1 - \theta \delta u^{-1}}{1 - \theta \delta} \iff h > \bar{h} := \frac{1}{\beta} - \left(\frac{\mu_\tau}{1 - \beta}\right)^{1 - u}$$

Notice that an RSE exists only if $h = \bar{h} = u\beta/\delta$. Thus, $\bar{h} < \bar{h}$ if and only if $\frac{\mu_\tau}{1 - \beta} > \left(\frac{1 - \beta - 1}{1 - \beta}\right)^{1 - \gamma^r}$. \(\Box\)

**Derivation of (32).** Any solution to the Pareto problem is characterized by the optimal accumulation of N-goods that maximizes the discounted sum of T-production

$$\max \{d_t\} \sum_{t=0}^{\infty} \delta^t d_t^o, \quad \text{s.t.} \quad k_{t+1} = \begin{cases} \theta k_t - d_t & \text{if } t \geq 1 \\
q_0 - d_0 & \text{if } t = 0 \end{cases}, \quad d_t \geq 0, q_0 \text{ given}$$

The Hamiltonian associated with this problem is $H_t = \delta^t[d_t]^\alpha + \lambda_t [\theta k_t - d_t]$. Since $\alpha \in (0, 1)$, the necessary and sufficient conditions for an optimum are

$$0 = H_d = \delta^t \alpha[d_t]^\alpha - \lambda_t, \quad \lambda_{t-1} = H_k = \theta \lambda_t, \quad \lim_{t \to \infty} \lambda_k t = 0$$

Thus, the Euler equation is

$$\hat{d}_{t+1} = [\delta \theta]^{1-\alpha} d_t = \theta \hat{d}_t, \quad \hat{\phi} := [\delta \theta]^{1-\alpha} \quad t \geq 1$$

To get a closed form solution for $d_t$ we replace (50) in the accumulation equation:

$$k_t = \theta^{t-1} k_1 - d_0 \sum_{s=0}^{t-2} \theta^{t-s-2} [\delta \theta]^{1-\alpha} = \theta^{t-1} \left[k_1 - d_0 \hat{\phi} \frac{1 - \hat{\phi}^{t-1}}{1 - \hat{\phi}} \right] = \theta^{t-1} \left[k_1 - \frac{d_1}{\theta} \frac{1 - \hat{\phi}^{t-1}}{1 - \hat{\phi}} \right]$$

Replacing (50) and (51) in the transversality condition we get

$$0 = \lim_{t \to \infty} \delta^t \alpha [d_t]^\alpha k_t = \lim_{t \to \infty} \delta^t \alpha \left[\left[\delta \theta \right]^{1-\alpha} d_0 \right]^\alpha \left[\theta^{t-1} k_1 - d_0 \hat{\phi} \frac{1 - \hat{\phi}^{t-1}}{1 - \hat{\phi}} \right] = \frac{\alpha d_0^{\alpha-1}}{\theta} \left[k_1 - d_0 \hat{\phi} \frac{1 - \hat{\phi}^{t-1}}{1 - \hat{\phi}} \right] \text{ iff } \hat{\phi} < 1$$

\(^{21}\) $E(1 + \gamma^r)$ is the geometric mean of $1 + \gamma^r, 1 + \gamma^k$ and $1 + \gamma^{nl}$. 

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Since \( k_1 = q_0 - d_0 \), the bracketed term equals zero if and only if \( \hat{d}_0 = [1 - \hat{o}]q_0 \). The accumulation equation then implies that the unique optimal solution is \( \hat{d}_t = [1 - \hat{o}]q_t \). □

**Derivation of (36).** To simplify notation we assume temporarily that there is only one crisis (at time \( \tau \)). It follows that profits and the bailout cost are:

\[
\pi_t = \frac{\alpha}{1 - \phi^t} \beta y_t - \frac{\alpha^t h}{u} y_{t-1}, \quad t \neq \{0, \tau, \tau + 1\} \\
\pi_0 = \frac{\alpha}{1 - \phi^0} \beta y_0, \quad \pi_\tau = 0, \quad \pi_{\tau + 1} = \frac{\alpha}{1 - \phi^1} \beta y_{\tau + 1} - \frac{\alpha \phi^c}{1 - \phi^c} h y_\tau
\]

(52)

\[T(\tau) = L_{\tau - 1} - \mu_p q_\tau = \frac{\alpha}{1 - \phi^c} h \beta y_{\tau - 1} - \mu_p q_\tau = \frac{\alpha}{1 - \phi^c} h \beta y_{\tau - 1} - \mu \frac{\alpha}{1 - \phi^c} y_\tau\]

(53)

Replacing these expressions in welfare function (34) and using the market clearing condition \( p_t q_t [1 - \phi_t] = \alpha y_t \), we get

\[
W(\tau) = (1 - \alpha) y_\tau + \frac{\alpha \beta y_\tau}{1 - \phi^c} + \frac{\alpha \beta y_\tau}{1 - \phi^c} - \frac{\alpha \phi^c}{1 - \phi^c} \frac{h}{u} y_{\tau - 1} + \delta^{\tau + 1} \left[ (1 - \alpha) y_{\tau + 1} + \frac{\alpha \beta y_{\tau + 1}}{1 - \phi^c} - \frac{\alpha \phi^c}{1 - \phi^c} \frac{h}{u} y_{\tau - 1} \right] \\
= \sum_{t \neq \tau} \delta^t \left[ (1 - \alpha) y_t + \alpha \frac{\beta}{1 - \phi^c} y_t - \frac{\alpha}{1 - \phi^c} \delta h \phi^c \frac{y_t}{u} \right] + \delta^\tau \left[ (1 - \alpha) y_\tau + \mu \frac{\alpha}{1 - \phi^c} y_\tau - \frac{\alpha \phi^c}{1 - \phi^c} \delta h \phi^c \right] \\
= \sum_{t \neq \tau} \delta^t y_t + K^c y_\tau, \quad K^c := 1 + \mu \frac{\alpha}{1 - \phi^c} - \frac{\alpha}{1 - \phi^c} \delta h \phi^c = 1 - \frac{\alpha [1 - (\mu + \mu_w)]}{1 - \phi^c}
\]

Notice that \( K^c \) can be simplified as follows

\[K^c = \alpha + \frac{\alpha}{1 - \phi^c} (\mu - (1 - \mu_w) + (1 - \mu_w) - \delta h \phi^c) = \alpha + \frac{\alpha}{1 - \phi^c} ((1 - \mu_w) - \delta h \phi^c) = \frac{\alpha [1 - (\mu + \mu_w)]}{1 - \phi^c} \]

Notice that \( \frac{1}{1 - \phi^c} ((1 - \mu_w) - \delta h \phi^c) = \frac{(1 - \mu_w)}{1 - \phi^c} \frac{(1 - \phi^c)}{1 - \phi^c} \mu_w = \frac{1 - \phi^c}{1 - \phi^c} = 1. \) Thus, \( K^c = 1 - \frac{\alpha [1 - (\mu + \mu_w)]}{1 - \phi^c} \). The expression for expected welfare in (36) follows by allowing multiple crises to take place.

**Derivation of (37).** Consider T-output net of bankruptcy costs: \( \bar{y}_t = K_t y_t \), where \( K_t \) is defined in (36).

Notice that \( W^r = E_0 \sum_{t = 0}^{\infty} \delta^t K_t y_t = E_0 \sum_{t = 0}^{\infty} \delta^t \bar{y}_t \), and \( \bar{y}_t \) follows a three-state Markov chain defined by:

\[
\tilde{T} = \begin{pmatrix}
0 & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0
\end{pmatrix}, \quad \tilde{G} = \begin{pmatrix}
g_1 \\
g_2 \\
g_3
\end{pmatrix} = \begin{pmatrix}
\frac{(\theta \phi^c)^\alpha K_u}{1 - \phi^c} \\
\frac{\theta \phi^c \beta u}{1 - \phi^c} \\
\frac{\theta \phi^c \beta u}{1 - \phi^c}
\end{pmatrix}
\]

(54)

To derive \( W^r \) in closed form consider the following recursion

\[
V(\bar{y}_0, g_0) = E_0 \sum_{t = 0}^{\infty} \delta^t \bar{y}_t = \bar{y}_0 + \delta E_0 V(\bar{y}_1, g_1) \\
V(\bar{y}_t, g_t) = y_t + \beta E_t V(\bar{y}_{t+1}, g_{t+1})
\]

(55)

Suppose that the function \( V \) is linear: \( V(\bar{y}_t, g_t) = \bar{y}_t w(g_t) \), with \( w(g_t) \) an undetermined coefficient. Substituting this guess into (55), we get \( w(g_t) = 1 + \delta E_t g_{t+1} w(g_{t+1}) \). Combining this condition with (54), it follows that \( w(g_{t+1}) \) satisfies

\[
\begin{pmatrix}
w_1 \\
w_2 \\
w_3
\end{pmatrix} = \begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix} + \delta \begin{pmatrix}
u & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0
\end{pmatrix} \begin{pmatrix}
g_1 w_1 \\
g_2 w_2 \\
g_3 w_3
\end{pmatrix} \Rightarrow \begin{pmatrix}
w_1 \\
w_2 \\
w_3
\end{pmatrix} = \begin{pmatrix}
\frac{1 + (1 - u) \delta g_2}{1 - (1 - u) \delta g_2} \\
\frac{1 + (1 - u) \delta g_2}{1 - (1 - u) \delta g_2} \\
\frac{1 + (1 - u) \delta g_2}{1 - (1 - u) \delta g_2}
\end{pmatrix}
\]

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This solution exists and is unique provided \( g_1 \delta u + g_2 g_3 \delta^2 (1 - u) < 1 \). Equation (37) follows by noting that at time 0 the economy is in the lucky state: \( V(y_0, g_0) = w_1 y_0 \), and by making the substitution \( g_2 g_3 = (\theta \phi^l)^\alpha (\theta \phi^c)^\alpha \).

**Proof of Proposition 4.2.** The welfare of a risky and a safe economy are given by (35) and (36), respectively. Clearly, if \( u = 1 \), both are equal. Since \( W^s \) does not depend on \( u \), we will prove the proposition by showing that when crises costs are small (i.e., \( \mu \to \beta \) and \( \mu_w \to 1 - \beta \), so that \( k_c \to 1 \)) the derivative \( W^u_\tau := \partial W^u / \partial u |_{u=1} \) is negative if and only if \( \phi^s < \phi^{po} \). Let us denote:

\[
L = 1 - \left[ \theta \phi^l \right]^\alpha \delta u - \left[ \theta^2 \phi^l \phi^c \right]^\alpha \delta^2 (1 - u), \quad T = \left( 1 + \delta (1 - u) \right) \left[ \theta \phi^l \frac{1 - \phi^s}{1 - \phi^c} \right]^\alpha (1 - \phi^l)^\alpha
\]

The derivatives of \( L \) and \( T \) evaluated at \( u = 1 \) are:

\[
L_u = -\delta (\theta \phi)^\alpha - \alpha \phi' \delta (\theta \phi)^\alpha - 1 + [\theta \phi]^\alpha \delta^2 \quad T_u = -\alpha \phi' (1 - \phi)^\alpha - 1 - \delta [\theta \phi]^\alpha (1 - \phi)^\alpha = (1 - \phi)^\alpha - 1 - \delta [\theta \phi]^\alpha (1 - \phi)
\]

where \( \phi = \phi^s \) and \( \phi' = \partial \phi^l / \partial u |_{u=1} \). Since \( W^u(u) = T/L \), it follows that

\[
\frac{T^2 W^u}{T^2} = (D - 1)(1 - \phi)^\alpha - 1 (\alpha \phi' + D (1 - \phi)) + (1 - \phi)^\alpha (D + \alpha \phi' \frac{D}{\phi^c} - D^2)
\]

where \( D = \delta (\theta \phi)^\alpha \). Since \( \phi < 1 \) and \( \phi' < 0 \), we have that \( W^u < 0 \) if and only if \( \delta (\theta \phi)^\alpha (1 - \phi)^\alpha - 1 > 1 \). Recall from (33) that the Pareto optimal share is \( \phi^{po} = (\theta \delta)^\frac{1}{1 - \alpha} \). Hence, we can rewrite this condition as \( W^u < 0 \) if and only if \( \phi^s < (\delta \theta )^{\frac{1}{1 - \alpha}} = \phi^{po} \). Since the system is continuous in \( u, \mu \) and \( \mu_w \), the result in the Proposition follows.

**Derivation of (39).** Suppose for a moment that there is only one crisis (at \( \tau \)). Then consumers welfare is

\[
C(\tau) = (1 - \alpha) y_c + \sum_{t \neq \tau} \delta^t (1 - \alpha) y_t + \delta^\tau [(1 - \alpha) y_{\tau} - T(\tau)]
\]

Using \( T(\tau) = \frac{\alpha}{1 - \phi} y_{\tau} - 1 - \mu \frac{\alpha}{1 - \phi} y_{\tau} \) and \( y_t = (\theta \phi^l)^\alpha \left[ \frac{1 - \phi^c}{1 - \phi} \right]^\alpha y_{t-1} \), it follows that

\[
(1 - \alpha) y_{\tau} - T(\tau) = y_{\tau} \left( 1 - \alpha - \frac{\alpha}{1 - \phi^c} \left[ \frac{h}{\alpha \theta^\alpha} \left[ \frac{1 - \phi^c}{1 - \phi} \right] \right]^{1 - \alpha} - \mu \right)
\]

\[
= (1 - \alpha) y_{\tau} \left( 1 - \frac{\alpha}{(1 - \phi^c)(1 - \alpha)} \left[ \frac{h}{\alpha \theta^\alpha} \left[ \frac{1 - \phi^c}{1 - \phi} \right] \right] - \mu \right) \equiv (1 - \alpha) y_{\tau} K^T_c
\]

If we allow multiple crises to occur, consumer’s welfare is

\[
C^c = (1 - \alpha) E_0 \sum_{t=0}^{\infty} \delta^t K_t y_t, \quad K_t = \begin{cases} 1 & \text{if } t \neq \tau_i \\ K^T_c & \text{if } t = \tau_i \end{cases}
\]

Following the same steps as in the derivation of (37) we get (39).
C. Model Simulations

The behavior of the model economy is determined by eight parameters: $u, r, \alpha, \theta, h, \beta, \mu_w$ and $\mu$. We will set the probability of crisis $1 - u$, the world interest rate $r$ and the share of N-inputs in T-production $\alpha$ equal to some empirical estimates. Then, given the values of $u, r$ and $\alpha$, we determine the feasible set for the degree of contract enforceability $h$ and the index of total factor productivity in the N-sector $\theta$ such that both an RSE and an SSE exist. The values of $\beta, \mu_w$ and $\mu$ are irrelevant for the existence of equilibria.

In a panel of 39 MECs studied in Tornell and Westermann (2002), the probability of a crisis in a given period ranges from 5% to 9%. The interest rate $r$, is set to the average US interest rate from 1980:1 to 1999:4, which equals 0.075. A survey of Mexican manufacturing firms suggests a conservative value for $\alpha$ equal to 35%. We then choose $\beta, \theta$ and $h$ so that: (i) both an RSE and an SSE exist for the range $u \in [0.91, 1]$, and (ii) we obtain plausible values for the growth rates along a safe economy and along a lucky path. In the baseline case: $h = 0.76, \theta = 1.65, \beta = 0.8$ and $u = 0.95$. These parameters imply a safe GDP growth rate of $(1 + \gamma_s) = (1 - \beta)^{\alpha \theta (1 - h)} = 3.8\%$ and a lucky GDP growth rate of $(1 + \gamma_l) = (1 - \beta)^{\alpha (1 - \mu_w (1 - u))^{1 - h}} = 8.7\%$. By comparison, the average growth rate of India over the period is 5.14% and that of Thailand is 8.14%.

We choose the financial distress costs of crises $l^d = 1 - \frac{\mu_w}{1 - \beta}$ so that the cumulative decrease of GDP during a crisis episode is 13%, which is the mean value in the sample considered by Tornell and Westermann (2002). In the model, the cumulative decrease in GDP growth during a crisis episode is $(1 + \gamma^{cr})^2 = \left(\frac{\mu_w}{1 - \beta}\right)^{\alpha (1 - \mu_w (1 - u))^{1 - h}}$. Using the baseline case $h = 0.76, \theta = 1.65, \alpha = 0.35$ we get that $(1 + \gamma^{cr})^2 = (1 - 0.13)$ if $\left(\frac{\mu_w}{1 - \beta}\right)^{\alpha (1 - \mu_w (1 - u))^{1 - h}} = 0.45$. Thus, we set conservatively $l^d = 0.7$. In the baseline case, the level of bankruptcy costs is free.

Finally, in order for the welfare measures to be bounded, the expected discounted sum of tradable production has to be finite. In the safe economy this requires $\delta (\theta \phi^s)^{\alpha} < 1$. In the risky economy: $\left[\theta \phi^l \right]^{\alpha} \delta u + \left[\theta^2 \phi^l \phi^c \right]^{\alpha} \delta^2 (1 - u) < 1$. These two conditions impose an upper bound on $\alpha$. In particular, they hold if $\alpha < 0.6$. Summing up:

\[\begin{align*}
22 & \text{Notice that the interior condition for the pareto optimal share, } \phi^{po} = \left[\theta \phi^s \right]^{\frac{1}{1-\alpha}} < 1 \text{ is sufficient for all boundness conditions if } \phi^l < \phi^{po}. \text{ This condition is equivalent to an upper bound on } \alpha : \alpha = \frac{\log(1 + r)}{\log(\theta)}. 
\end{align*}\]
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